DOCUMENT RESUME

ED 281 866 TM 870 252

AUTHOR	Kirby, Peggy C.
TITLE	Confirmatory Factor Extraction Versus Confirmatory Factor Rotation.
PUB DATE	30 Jan 87
NOTE	21p.; Paper presented at the Annual Meeting of the Southwest Educational Research Association (Dallas, TX, January 30, 1987).
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Correlation; Error of Measurement; *Factor Analysis; *Factor Structure; Goodness of Fit; *Hypothesis Testing; Mathematical Models; Matrices; *Maximum Likelihood Statistics; Statistical Studies
IDENTIFIERS	Chi Square; *Confirmatory Factor Analysis; *Exploratory Factor Analysis; LISREL Computer Program; Residuals (Statistics)

ABSTRACT

Confirmatory factor analysis is a true statistical technique in the sense that it can test hypothesized relationships among variables. Unlike exploratory factor analysis, confirmatory factor analysis can be used to disconfirm or support a priori expectations about the target population based on sample data. Confirmatory factor analysis can be combined with exploratory techniques in theory building. Such is the case in confirmatory factor extraction with subsequent relaxation of parameter constraints in stepwise fashion to determine the best model. Initial exploratory factor extraction can also be followed by confirmatory factor rotation. Confirmatory factor extraction differs from confirmatory factor rotation with respect to the nature of the hypothesized relationships and the point at which the hypotheses are tested. Results of both confirmatory factor extraction and confirmatory factor rotation must be interpreted with caution. Assumptions of the techniques should not be violated, and magnitude of residuals should be examined as an estimate of the extent to which the model is capitalizing on error variance. A hypothetical data set (scores of 50 subjects on nine variables) is used to empirically demonstrate these methods. (JAZ)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ERIC

ED281866

Confirmatory Factor Extraction
versus
Confirmatory Factor Rotation

Peggy C. Kirby

University of New Orleans 70148

Paper presented at the annual meeting of the Southwest Educational Research Association, Dallas, TX, January 30, 1987

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

P. C. Kirby

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

- This document has been reproduced as received from the person or organization originating it.
- Minor changes have been made to improve reproduction quality.
- Points of view or opinions stated in this document do not necessarily represent official OERI position or policy

TM 870 252

BEST COPY AVAILABLE

Confirmatory Factor Extraction
versus
Confirmatory Factor Rotation

Peggy C. Kirby

University of New Orleans

The most common applications of factor analytic techniques in the behavioral sciences are exploratory methods wherein the number of factors and the relationships among variables are entirely determined by the data being analyzed (Nunnally, 1978). More recent confirmatory factor analytic techniques, on the other hand, are based on a priori assumptions about characteristics of the population and seek to generalize from sample data about the population parameters (Joreskog, 1969).

Ironically, factor analysis was originally conceptualized as a confirmatory process. Spearman hypothesized that all tests of mental abilities measured one general factor (G). By subtracting a matrix of cross-products of the structure coefficients of G from the matrix of correlations, a residual matrix is obtained which can be examined to determine the tenability of the general factor hypothesis.

It was quickly realized that Spearman's G factor methods would, in most cases, yield large residual coefficients and thus suggest a multifactor case. Subsequent techniques were based on the extraction of a general factor and the examination of the residual matrix to determine additional factors. Since groupings were determined by the residuals rather than by hypothesized relationships, Spearman's factor analytic model, developed for hypothesis confirmation, became an exploratory technique (Nunnally, 1978).

In confirmatory factor analysis, the researcher has a conceptual basis for hypothesizing patterns of relationships in the sample data. Hypothesized relationships may involve the number of factors, the factor structure coefficients for some or all variables, or the correlations among factors. Hypotheses must have theoretical or empirical support. A series of exploratory factor analyses can eventually yield enough evidence to suggest relationships that can be tested using confirmatory analysis.

Confirmatory Factor Analysis

Confirmatory factor extraction differs from confirmatory factor rotation (Thompson, 1986) with respect to the nature of the hypothesized relationships

and the point at which the hypotheses are tested. In confirmatory extraction, the researcher may confirm or disconfirm conclusions regarding the hypothesized number of factors, the correlations among factors, or the correlations between variables and factors (i.e., structure coefficients). The three hypotheses are supported, respectively, by the magnitudes of the residuals, the inter-factor correlations, and the magnitude of the structure coefficient of each variable on its predicted factor.

Joreskog (1969) developed a method of confirmatory factor extraction which applies a direct solution to the correlation matrix. Since confirmatory extraction is driven by hypothesized factor structure, the solution is directly interpretable. Rotation is not usually required. Maximum likelihood extraction employs a target matrix that identifies which population parameters are to be estimated. LISREL VI is a computerized mathematical package that performs maximum likelihood factor analysis and provides chi-square estimates of the goodness of fit of the sample data to the hypothesized structure (Joreskog, 1973).

Thompson (1986) suggests that several caveats and assumptions apply to confirmatory factor extraction

methods. It is assumed that the data are multivariate normally distributed although the procedures have been shown to be somewhat robust to violations of this assumption. In addition, a sample of at least 50 replicates is required, perhaps more if there are a large number of factored entities. As in all statistical applications, it is assumed that the sample is representative of the population. Constraints placed on parameters must have a strong theoretical basis. The methods are not readily applicable in the absence of rational expectations regarding outcomes.

Confirmatory Factor Rotation

If confirmatory factor extraction is not feasible, factors may be extracted using one of the more common exploratory techniques and the resulting factor matrix rotated to a position of best fit with the hypothesized matrix. Kaiser, Hunka, and Bianchini (1969) devised a method of "relating" sets of factors. Two sets of factors obtained from different replicates are projected into the same space and the cosines of the angles among the factors across the two solutions are computed. The cosines are actually correlation coefficients since factors are standardized to unit length (Gorsuch, 1983). Similarly, a set of obtained

factors may be rotated to a position of best fit with a hypothesized target matrix (Thompson, 1986). A computer program developed by Veldman (1967) to perform this rotation process is used in the sample analysis in the present study.

Sample Analysis

A simple hypothetical data set was created to empirically demonstrate the processes of confirmatory factor extraction and confirmatory factor rotation. Scores of 50 subjects on nine variables are presented in Table 1. Assuming that there is adequate theoretical or empirical support for hypothesizing relationships among variables and assuming that the sample is representative of the population and is multivariate normal, the researcher may employ confirmatory factor analytic techniques.

Table 2 presents the parameter specifications used in the example LISREL model. The researcher anticipates that the data are best explained by three uncorrelated factors with variables 1, 2, and 9 correlating only with Factor I; variables 3, 4, and 5 with Factor II; and variables 6, 7, and 8 with Factor III. The extraction process yields the directly interpretable solution presented in Table 3. All

maximum likelihood estimates of the correlations between original variables and their hypothesized factors exceed .67 with the exception of variable 2 which has an estimated structure coefficient of only .26. Standard errors range from .13 to .29, with the variables hypothesized to constitute Factor I having the largest standard errors.

The chi-square goodness of fit test (Table 4) compares the model obtained under the imposed constraints to an unconstrained model. A chi-square value that is large relative to degrees of freedom indicates a poor model. The chi-square of 30.29 and adjusted goodness of fit index of .82 for this three-factor model can be compared to estimates of a hypothesized two-factor model presented in Tables 5 and 6. The two-factor model yields a statistically significant ($p<.05$) chi-square of 43.76 with an adjusted goodness of fit index of .74.

In addition to the information presented above, LISREL VI calculates modification indices which Joreskog and Sorbom proposed as the decrease in chi-square to be expected if any single parameter constraint is relaxed (Long, 1983). Modification indices for variables under the three-factor model are

presented in Table 7. The most improvement in the model (i.e., most decrease in chi-square) would be obtained by releasing the constraint that variable 2 have a zero correlation with Factor III. The modification index of 8.43 suggests a non-zero correlation.

Relaxing parameter constraints one by one based on magnitude of modification indices does become an exploratory rather than a confirmatory technique. The appropriate factor structure matrix is now being determined by the data rather than being derived from theory. Conversely, confirmatory factor rotation begins with exploratory analysis of the data, proceeding to confirm hypothesized relationships only after factors are determined by the sample data.

The raw data presented in Table 1 were subjected to a principal components factor analysis with varimax rotation. The rotated structure matrix is presented in Table 8. This matrix was rotated to a position of best fit with the target matrix presented in Table 9 using Veldman's (1967) "RELATE" program. The target matrix specifies the same relationships hypothesized in the confirmatory extraction example.

The projection of the two matrices into the same

factor space yields the structure matrix presented in Table 10. Cosines of angles between factors of the two sets are included in Table 11. Diagonal entries indicate that correlations between like factors range from .96 to 1.00. These correlations appear to be statistically significant ($p<.05$) based on Thompson's (1986) partial test distribution for factor cosines.

While the "relate" procedure is useful as an invariance technique and in instrument validation, it is criticized for its tendency to capitalize on sampling error (Nunnally, 1978; Thompson, 1986). The larger item-factor correlations produced by confirmatory rotation (Table 10) as compared to those resulting from confirmatory extraction (Table 3) support this criticism, although smaller structure coefficients would have resulted if principal factor analysis had been employed rather than principal components analysis.

Summary

Confirmatory factor analysis is a true statistical technique in the sense that it can test hypothesized relationships among variables. Though intuitively appealing to the novice researcher (Cronkhite & Liska, 1980), exploratory factor analytic techniques are

ERIC

criticized for their employment "without due regard for the value of ... research proposals" (Sax, 1979, p. 80). Unlike exploratory factor analysis, confirmatory factor analysis can be used to disconfirm or support a priori expectations about the target population based on sample data.

Confirmatory factor analysis can be combined with exploratory techniques in theory building. Such is the case in confirmatory extraction with subsequent relaxation of parameter constraints in stepwise fashion to determine the best model. Initial exploratory factor extraction can also be followed by confirmatory rotation.

Results of both confirmatory factor extraction and confirmatory factor rotation must be interpreted with caution. Assumptions of the techniques should not be violated, and magnitude of residuals should be examined as an estimate of the extent to which the model is capitalizing on error variance.

References

Cronkhite, G. & Liska, J. R. (1980). The judgment of communicant acceptability. In M.E. Roloff & G.R. Miller (Eds.), Persuasion: New directions in theory and research. Beverly Hills: Sage, pp. 101-139.

Gorsuch, R. (1983). Factor analysis (2nd ed.). Hillsdale, NJ: Erlbaum.

Joreskog, K. G. (1969). A general approach to confirmatory maximum likelihood factor analysis. Psychometrika, 34, 183-202.

Joreskog, K. G. (1973). A general model for estimating a linear structural equation system. In A.S. Goldberger & O.D. Duncan (Eds.), Structural equation models in the social sciences, New York: Seminar Press.

Kaiser, H. F., Hunka, S. & Bianchini, J. (1969). Relating factors between studies based upon different individuals. In H.J. Eysenck & S.B.G. Eysenck (Eds.), Personality structure and measurement. San Diego: Knapp, pp. 333-343.

Long, J. S. (1983). Confirmatory factor analysis. Beverly Hills, CA: Sage Publications.

Nunnally, J. C. (1978). Psychometric theory. New York: McGraw-Hill.

ERIC

Sax, G. (1979). Foundations of educational research (2nd ed.). Englewood Cliffs: Prentice-Hall.

Thompson, B. (1986). A partial test distribution for cosines among factors across samples. Paper presented at the annual meeting of the American Educational Research Association, San Francisco.

Veldman, D. J. (1967). FORTRAN programming for the behavioral sciences. New York: Holt, Rinehart & Winston.

Table 1

Hypothetical Data Set (9 Variables/ 50 Replicates)

114554332	224545223	554453344
445452333	235454452	334534544
235553111	125353332	334345214
334542242	335552233	434433233
224334543	554541214	544543114
515554443	445452323	443441315
223321344	234451214	341122312
333543333	244532223	242334212
153415313	334453323	512225544
411124454	224352322	352322133
234555453	123454341	352212224
234543232	454435453	453234244
234542122	543224344	532125555
214544443	324453232	234541113
125341111	232412235	231221212
323324342	231122124	323433533
131454442	152331211	

Table 2

Parameters To Be Estimated in LISREL 3-Factor Model

Variable	Factor 1	Factor 2	Factor 3
1	1	0	0
2	2	0	0
3	0	3	0
4	0	4	0
5	0	5	0
6	0	0	6
7	0	0	7
8	0	0	8
9	9	0	0

Table 3

LISREL Estimates and Standard Errors
3-Factor Model

LISREL ESTIMATES (MAXIMUM LIKELIHOOD)

LAMBDA X

	FACTOR1	FACTOR2	FACTOR3
T1-1	0.837	0.0	0.0
T2-2	0.256	0.0	0.0
D1-3	0.0	0.878	0.0
D2-4	0.0	0.756	0.0
D3-5	0.0	0.781	0.0
P1-6	0.0	0.0	0.677
P2-7	0.0	0.0	0.813
P3-8	0.0	0.0	0.804
T3-9	0.711	0.0	0.0

STANDARD ERRORS

LAMBDA X

	FACTOR1	FACTOR2	FACTOR3
T1-1	0.294	0.0	0.0
T2-2	0.163	0.0	0.0
D1-3	0.0	0.127	0.0
D2-4	0.0	0.131	0.0
D3-5	0.0	0.130	0.0
P1-6	0.0	0.0	0.139
P2-7	0.0	0.0	0.137
P3-8	0.0	0.0	0.137
T3-9	0.261	0.0	0.0

Table 4

MEASURES OF GOODNESS OF FIT FOR THE WHOLE MODEL :

CHI-SQUARE WITH 27 DEGREES OF FREEDOM IS 30.29 (PROB. LEVEL = 0.301)

GOODNESS OF FIT INDEX IS 0.890

ADJUSTED GOODNESS OF FIT INDEX IS 0.816

ROOT MEAN SQUARE RESIDUAL IS 0.132

Table 5

Parameter Specifications and Estimates 2-Factor Model

PARAMETER SPECIFICATIONS

LAMBDA X

	FACTOR1	FACTOR2
T1-1	1	0
T2-2	2	0
D1-3	0	3
D2-4	0	4
D3-5	0	5
P1-6	6	0
P2-7	7	0
P3-8	8	0
T3-9	9	0

LISREL ESTIMATES (MAXIMUM LIKELIHOOD)

LAMBDA X

	FACTOR1	FACTOR2
T1-1	0.260	0.0
T2-2	-0.356	0.0
D1-3	0.0	0.878
D2-4	0.0	0.756
D3-5	0.0	0.781
P1-6	0.665	0.0
P2-7	0.829	0.0
P3-8	0.800	0.0
T3-9	0.213	0.0

Table 6

MEASURES OF GOODNESS OF FIT FOR THE WHOLE MODEL :

CHI-SQUARE WITH 27 DEGREES OF FREEDOM IS 43.76 (PROB. LEVEL = 0.022)

GOODNESS OF FIT INDEX IS 0.841

ADJUSTED GOODNESS OF FIT INDEX IS 0.736

ROOT MEAN SQUARE RESIDUAL IS 0.134

ERIC

Table 7

Modification Indices

3-Factor Model

MODIFICATION INDICES

LAMBDA X

	FACTOR1	FACTOR2	FACTOR3
T1-1	0.0	1.105	2.249
T2-2	0.0	0.671	8.425
D1-3	1.457	0.0	0.057
D2-4	0.780	0.0	0.523
D3-5	2.282	0.0	0.009
P1-6	0.066	0.033	0.0
P2-7	0.507	0.473	0.0
P3-8	0.475	0.075	0.0
T3-9	0.0	4.166	0.314

ERIC

Table 8

VARIMAX Rotated Structure Matrix

1	0.16025	0.02854	0.87773
2	-0.56698	-0.11555	0.47572
9	0.10392	-0.22959	0.81956
3	0.01345	0.91025	0.01817
4	-0.07860	0.84353	-0.07002
5	0.04187	0.85123	-0.21205
6	0.[illegible]105	-0.00918	0.05628
7	0.85342	-0.08006	0.13432
8	0.84060	-0.00134	0.14180

Table 9

Target Structure Matrix

0.00000	0.00000	1.00000
0.00000	0.00000	1.00000
0.00000	0.00000	1.00000
0.00000	1.00000	0.00000
0.00000	1.00000	0.00000
0.00000	1.00000	0.00000
1.00000	0.00000	0.00000
1.00000	0.00000	0.00000
1.00000	0.00000	0.00000

Table 10

Rotated Factor Matrix

0.3146	0.0243	0.8510
-0.4483	-0.0908	0.5390
0.2652	-0.2385	0.8060
-0.0377	0.9347	-0.0055
-0.1383	0.8702	-0.0808
-0.0497	0.8720	-0.2356
0.7499	-0.0454	-0.0320
0.8475	-0.1219	0.0376
0.8319	-0.0404	0.0446

Table 11

Cosines of Angles Between Factors

	1	2	3
1	0.96	-0.06	0.19
2	-0.05	1.00	0.00
3	-0.11	-0.02	0.99